Decay of massive scalar hair in the background of a black hole with a global mononpole

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Abstract

The late-time tail behaviors of massive scalar fields are examined analytically in the background of a black hole with a global monopole. It is found that the presence of a solid deficit angle in the background metric makes the massive scalar fields decay faster in the intermediate times. However, the asymptotically late-time tail is not affected and it has the same decay rate of $t^{-5/6}$ as in the Schwarzschild and nearly extreme Reissner-Nordström backgrounds.

PACS numbers: 04.20.Ex, 04.70.Bw

I. INTRODUCTION

Ever since Wheeler put forward the no-hair theorem [1], which states that the external field of a black hole relaxes to a Kerr-Newman type characterized solely by the black-hole's mass, charge and angular momentum, there have been a lot of investigations concerning the dynamical mechanism by which perturbations fields outside a black hole are radiated away. The massless scalar, gravitational and electromagnetic external perturbations were first studied by Price [2] in the Schwarzschild background and an inverse power-law tail, t^{2l+3}, has been found to dominate the late-time behavior of these perturbations for a fixed position, if there is no initial static field. Here l is the multiple moment of the wave mode and t is the Schwarzschild time coordinate. The late-time behaviors of these massless neutral perturbations along the null infinity and along the future event horizon were further examined by Gundlach et al [3,4]. Recently the late-time tail has also been considered in the case of a rotating black hole by Barack and Ori [5].

Although these works are mainly concerned with massless fields, the evolution of massive scalar fields is also important and it has attracted a lot of attention recently. Behaviors qualitatively different from those of massless fields have been found. For instance, It has been shown in Ref. [6] that an oscillatory power-law tail of the form $\sim t^{-l-\frac{3}{2}}\sin(\mu t)$ for massive scalar fields develops at the intermediate late-time characterized by $\mu M \ll 1$ in Reissner-Nordström background. Here μ is the mass of the scalar field and M is that of the black hole. Note that the massive scalar fields decay slower than massless ones. It should be pointed out, however, that this intermediate tail is not the final pattern that dominates at very late times [6]. In fact, a transition from the intermediate behavior to an oscillatory tail with the decay rate of $t^{-5/6}$ has been demonstrated to occur at asymptotically late times both in the Schwarzschild and nearly extreme Reissner-Nordström backgrounds [7,8].

In this paper, we will examine both the intermediate and asymptotic late time behaviors for massive scalar fields at a fixed radius in the background of a black hole with a global monopole. A global monopole is one of the topological defects that may have been formed during phase transitions in the evolution of the early universe and a black hole with a global monopole is the result of an interesting process in which a black hole swallows a global monopole [9]. An unusual property of the black-hole-global-monopole system is that it possesses a solid deficit angle, which makes it quite different topologically from that of a Schwarzschild black hole alone. The physical properties of the black-hole-global-monopole system have been studied extensively in recent years. These include, but are not limited to, the gravitational [9,10] and the vacuum polarization effects [11,12], the particle creation in the formation of the system [13], the black hole thermodynamics [14], and more recently the energy spectra of non-relativistic quantum system in the background [15].

Our purpose here is to see what effects the solid deficit angle in the background metric due to the presence of a global monopole will have on the late-time evolution of massive scalar fields. In Sec. II we describe the physical system and formulate the problem in terms of the black-hole Green's function using the spectral decomposition method [16]. In Sec. III, we examine both the intermediate and asymptotic late-time behaviors of massive scalar fields. We conclude in Sec. IV with a brief summary and some discussions.

II. DESCRIPTION OF THE SYSTEM AND GREEN'S FUNCTION FORMALISM

We examine the time evolution of a massive scalar field in the background of a black hole with a global monopole. The metric is described by

$$ds^2 = -\Big(1 - 8\pi G\eta_0^2 - \frac{2Gm}{r}\Big)\, dt^2 + \Big(1 - 8\pi G\eta_0^2 - \frac{2Gm}{r}\Big)^{-1} dr^2 + r^2 d\Omega^2 \ , \tag{1}$$

where m is the mass of the black hole and η_0 is the symmetry breaking scale when the monopole is formed [9] . Introducing the coordinate transformation

$$t \longrightarrow (1 - 8\pi G\eta_0^2)^{\frac{1}{2}} t, \qquad r \longrightarrow (1 - 8\pi G\eta_0^2)^{\frac{-1}{2}} r \tag{2}$$

and new parameters

$$M = (1 - 8\pi G\eta_0^2)^{-3/2} m, \qquad b = 1 - 8\pi G\eta_0^2 \tag{3}$$

then we can rewrite metric Eq. (1) as

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 b(d\theta^2 + \sin^2\theta d\phi^2) \tag{4}$$

This metric is, apart from the deficit solid angle $\Delta = 4\pi b = 32\pi G\eta_0^2$, very similar to the Schwarzschild metric and we will use this form thereafter. The equation of motion for a minimally coupled scalar field with mass μ is

$$\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu}\Big(\sqrt{-g} g^{\mu\nu} \frac{\partial \phi}{\partial x^\nu}\Big) - \mu^2 \phi = 0 \ . \tag{5}$$

ϕ can be separated in the given metric as

$$\phi = \sum_{l,m} \frac{\psi^l(r)}{r} Y_{lm}(\theta, \varphi), \tag{6}$$

hereafter we omit the index l of ψ^l for simplicity. Using the tortoise coordinate r_* defined by

$$dr_* = \frac{dr}{1 - \frac{2M}{r}} \ , \tag{7}$$

we obtain a wave equation for each multiple moment:

$$\psi_{,tt} - \psi_{,r_* r_*} + V\psi = 0, \tag{8}$$

where the effective potential V is

$$V = \left(1 - \frac{2M}{r}\right)\left[\frac{l(l+1)b^{-1}}{r^2} + \frac{2M}{r^3} + \mu^2\right]. \tag{9}$$

Define the retarded Green's function $G(r_*, r_*'; t)$ by

$$\left[\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial r_*^2} + V\right] G(r_*, r'_*; t) = \delta(t)\delta(r_* - r'_*) \ , \tag{10}$$

for $t > 0$. The causality condition gives the initial condition that $G(r_*, r'_*; t) = 0$ for $t \leq 0$. Then the time evolution of the massive scalar field is given by

$$\psi(r_*, t) = \int \left[G(r_*, r'_*; t)\psi_t(r', 0) + G_t(r_*, r'_*; t)\psi(r'_*, 0)\right] dr'_* \tag{11}$$

In order to find $G(r_*, r'_*; t)$ we use the Fourier transform

$$\tilde{G}(r_*, r'_*; \omega) = \int_{0^-}^{+\infty} G(r_*, r'_*; t)e^{i\omega t} dt. \tag{12}$$

The Fourier transform is analytic in the upper half ω plane, and the corresponding inversion formula is

$$G(r_*, r'_*; t) = -\frac{1}{2\pi}\int_{-\infty+ic}^{\infty+ic} \tilde{G}(r_*, r'_*; \omega)e^{-i\omega t} d\omega \tag{13}$$

where c is some positive constant.

Let $\tilde{\psi}_1(r_*, \omega)$ and $\tilde{\psi}_2(r_*, \omega)$ be two linearly independent solutions to the homogenous equation

$$\left(\frac{d^2}{dr_*^2} + \omega^2 - V\right)\tilde{\psi}_i = 0 \quad i = 1, 2. \tag{14}$$

The Green's function can be constructed as follows

$$\tilde{G}(r_*, r'_*; \omega) = -\frac{1}{W(\omega)} \begin{cases} \tilde{\psi}_1(r'_*, \omega)\tilde{\psi}_2(r_*, \omega) & , \qquad r'_* > r_*, \\ \tilde{\psi}_1(r_*, \omega)\tilde{\psi}_2(r'_*, \omega) & , \qquad r'_* < r_*. \end{cases} \tag{15}$$

Here $W(\omega)$ is the Wronskian defined as

$$W(\omega) = \tilde{\psi}_1\tilde{\psi}_{2,r_*} - \tilde{\psi}_{1,r_*}\tilde{\psi}_2. \tag{16}$$

To calculate $G(r_*, r'_*; t)$ using Eq. (13), one needs to close the contour of integration into the lower half of the complex frequency plane. It has been argued that the asymptotic tail is associated with the existence of a branch cut (in $\tilde{\psi}_2$) placed along the interval $-m \leq \omega \leq m$ [16,6]. This tail arises from the integral of $\tilde{G}(r_*, r'_*; \omega)$ around the branch cut (denoted by G^C) which gives rise to an oscillatory inverse power-law behavior of the field. Therefore our goal is to evaluate $G^C(r_*, r'_*; \omega)$.

III. EVOLUTION OF MASSIVE SCALAR FIELDS

Now let us assume that both the observer and the initial data are situated far away from the black-hole such that $r \gg M$. We expand the wave-equation (14) in M/r to obtain (neglecting terms of order $O[(\frac{M}{r})^2]$ and higher)

$$\left[\frac{d^2}{dr^2}+w^2-\mu^2+\frac{4Mw^2-2M\mu^2}{r}-\frac{l(l+1)b^{-1}}{r^2}\right]\xi=0\ , \tag{17}$$

where $\xi=(1-\frac{2M}{r})^{1/2}\tilde{\psi}$. This equation can be solved in terms of Whittaker's functions. The two basic solutions needed to construct the Green's function are (for $|w|\leq m$)

$$\tilde{\psi}_1=M_{\kappa,\rho}(2\varpi r)\ , \tag{18}$$

and

$$\tilde{\psi}_2=W_{\kappa,\rho}(2\varpi r)\ , \tag{19}$$

where

$$\varpi=\sqrt{\mu^2-\omega^2},\qquad \kappa=\frac{3}{2}\frac{M\mu^2}{\varpi}-2M\varpi,\qquad \rho=\sqrt{l(l+1)b^{-1}+\frac{1}{4}}. \tag{20}$$

Let us note that these solutions can also be written in terms of two standard confluent hypergeometric functions, $M(a,b,z)$ and $U(a,b,z)$, as follows,

$$\tilde{\psi}_1=M_{\kappa,\rho}(2\varpi r)=e^{-\varpi r}(2\varpi r)^{\frac{1}{2}+\rho}\ M(\rho+\frac{1}{2}-\kappa,\ 1+2\rho,\ 2\varpi r)\ , \tag{21}$$

and

$$\tilde{\psi}_2=W_{\kappa,\rho}(2\varpi r)=e^{-\varpi r}(2\varpi r)^{\frac{1}{2}+\rho}\ U(\rho+\frac{1}{2}-\kappa,\ 1+2\rho,\ 2\varpi r)\ , \tag{22}$$

Using Eq. (13), one finds that the branch cut contribution to the Green's function is given by

$$\begin{aligned}G^C(r'_*,r_*;t)&=\frac{1}{2\pi}\int_{-\mu}^{\mu}\left[\frac{\tilde{\psi}_1(r'_*,\varpi e^{\pi i})\tilde{\psi}_2(r_*,\varpi e^{\pi i})}{W(\varpi e^{\pi i})}-\frac{\tilde{\psi}_1(r'_*,\varpi)\tilde{\psi}_2(r_*,\varpi)}{W(\varpi)}\right]e^{-iwt}dw\\&=\frac{1}{2\pi}\int_{-\mu}^{\mu}f(\varpi)e^{-iwt}dw\ .\end{aligned} \tag{23}$$

For simplicity we assume that the initial data has a considerable support only for r-values which are smaller than the observer's location. This, of course, does not change the late-time behavior. Let us note that when t is large, the term e^{-iwt} oscillates rapidly. This leads to a mutual cancellation between the positive and the negative parts of the integrand, so that the effective contribution to the integral arises from $|w|=O(\mu-\frac{1}{t})$ or equivalently $\varpi=O(\sqrt{\frac{\mu}{t}})$ [6].

Using the following relations

$$\begin{aligned}W_{\kappa,\,\rho}(2\varpi r)=&\frac{\Gamma(-2\rho)}{\Gamma(\frac{1}{2}-\rho-\kappa)}\ M_{\kappa,\,\rho}(2\varpi r)\\&+\frac{\Gamma(2\rho)}{\Gamma(\frac{1}{2}+\rho-\kappa)}\ M_{\kappa,\,-\rho}(2\varpi r)\ ,\end{aligned} \tag{24}$$

and

$$M_{\kappa,\,\rho}(e^{\pi i}2\varpi r)=e^{(\frac{1}{2}+\rho)\pi i}M_{-\kappa,\,\rho}(2\varpi r)\ , \tag{25}$$

we find, with the help of 13.1.20 of Ref. [17], that

$$W(\varpi e^{\pi i})=-W(\varpi)=\frac{\Gamma(2\rho)}{\Gamma(\frac{1}{2}+\rho-\kappa)}\,4\rho\varpi\ , \tag{26}$$

and consequently,

$$\begin{aligned} f(\varpi)=&\frac{1}{4\rho\varpi}\ [M_{\kappa,\,\rho}(2\varpi r'_*)M_{\kappa,\,-\rho}(2\varpi r_*)-M_{-\kappa,\,\rho}(2\varpi r'_*)M_{-\kappa,\,-\rho}(2\varpi r_*)] \\ &+\frac{1}{4\rho\varpi}\frac{\Gamma(-2\rho)\Gamma(\frac{1}{2}+\rho-\kappa)}{\Gamma(2\rho)\Gamma(\frac{1}{2}-\rho-\kappa)}\Big[M_{\kappa,\,\rho}(2\varpi r'_*)M_{\kappa,\,\rho}(2\varpi r_*) \\ &+e^{(2\rho+1)\pi i}\ M_{-\kappa,\,\rho}(2\varpi r'_*)M_{-\kappa,\,\rho}(2\varpi r_*)\Big]\ . \end{aligned} \tag{27}$$

A. Intermediate late-time Tails

First we discuss the intermediate asymptotic behavior of the massive scalar field. That is the tail in the range

$$M\ll r\ll t\ll\frac{M}{(\mu M)^2}\ . \tag{28}$$

In this time scale, the frequency range $\varpi=O(\sqrt{\frac{\mu}{t}})$, which gives the dominant contribution to the integral, implies

$$\kappa\ll 1\ . \tag{29}$$

Notice that κ originates from the $1/r$ term in the massive scalar field equation. It describes the effect of backscattering off the spacetime curvature. If the relation Eq. (29) is satisfied, the backscattering off the curvature from asymptotically far regions (which dominates the tails of massless fields) is negligible. However, it is worthwhile to point out that the intermediate time tail here will be different from that in the case without a global monopole because of the nontrivial topology in the background metric , i.e. , the presence of the solid deficit angle since $b\neq 1$. So, we have in this case,

$$f(\varpi)\approx\frac{(1+e^{(2\rho+1)\pi i})}{4\rho\varpi}\frac{\Gamma(-2\rho)\Gamma(\frac{1}{2}+\rho)}{\Gamma(2\rho)\Gamma(\frac{1}{2}-\rho)}M_{0,\,\rho}(2\varpi r'_*)M_{0,\,\rho}(2\varpi r_*)\ . \tag{30}$$

Since $\varpi r\ll 1$, the above equation can be further approximated, by using $M(a,b,z)\approx 1$ as $z\to 0$, to give

$$f(\varpi)\approx\frac{(1+e^{(2\rho+1)\pi i})\Gamma(-2\rho)\Gamma(\frac{1}{2}+\rho)}{4\rho\Gamma(2\rho)\Gamma(\frac{1}{2}-\rho)2^{-2\rho-1}}(r'_*r_*)^{\frac{1}{2}+\rho}\varpi^{2\rho}\ . \tag{31}$$

Substituting the above result into Eq. (23), we obtain

$$G^C(r'_*, r_*; t) = \frac{(1+e^{(2\rho+1)\pi i})\Gamma(-2\rho)\Gamma(\frac{1}{2}+\rho)}{8\pi\rho\Gamma(2\rho)\Gamma(\frac{1}{2}-\rho)2^{-2\rho-1}}(r'_* r_*)^{\frac{1}{2}+\rho} \int_{-\mu}^{\mu} \varpi^{2\rho}\, e^{-iwt}$$
$$= \frac{(1+e^{(2\rho+1)\pi i})\Gamma(-2\rho)\Gamma(\frac{1}{2}+\rho)\Gamma(\rho+1)\mu^{\rho+\frac{1}{2}}}{\sqrt{\pi}\rho\Gamma(2\rho)\Gamma(\frac{1}{2}-\rho)2^{-3\rho-\frac{3}{2}}}(r'_* r_*)^{\frac{1}{2}+\rho}\, t^{-\rho-\frac{1}{2}} J_{\rho+\frac{1}{2}}(\mu t) \;, \quad (32)$$

where $J_{\rho+\frac{1}{2}}$ is the Bessel function. In the limit $t \gg \mu^{-1}$, it becomes

$$G^C(r'_*, r_*; t) = \frac{(1+e^{(2\rho+1)\pi i})\Gamma(-2\rho)\Gamma(\frac{1}{2}+\rho)\Gamma(\rho+1)\mu^{\rho}}{\pi\rho\Gamma(2\rho)\Gamma(\frac{1}{2}-\rho)2^{-3\rho-2}}(r'_* r_*)^{\frac{1}{2}+\rho}\, t^{-\rho-1} \cos(\mu t - (\rho+1)\pi/2) \;, \quad (33)$$

which clearly exhibits an oscillatory inverse power-law behavior. Let's note that in general $b < 1$ and recall that

$$\rho = \sqrt{l(l+1)b^{-1} + \frac{1}{4}} \;, \quad (34)$$

then a comparison of the result here with Eq. (32) of Ref. [6] tells us that in the intermediate times the power-law tail depends not only on the multiple number of the wave mode but also on the the parameter (b) characterizing the space-time metric, and the massive scalar field decays faster in the black hole background with a global monopole than in that without it. So, although the intermediate tail is not affected significantly by the curvature, it is by the topology of the background metric.

B. asymptotic late-time tails

In the above calculation, we have used the approximation of $\kappa \ll 1$, which only holds when $\mu t \ll 1/\mu^2 M^2$. Therefore, the power-law tail found in the last section is not the final one, and a change to a different pattern of decay is expected when κ is not negligibly small. In this section, we examine the asymptotic tail behavior at very late times such that

$$\mu t \gg \frac{1}{\mu^2 M^2}. \quad (35)$$

Now we have

$$\kappa \simeq \frac{3M\mu^2}{2\varpi} \gg 1 \;. \quad (36)$$

So the backscattering off the curvature will be important in this case. Using Eq. (13.5.13) of Ref. [17], we have, in the limit $\kappa \gg 1$, that

$$M_{\pm\kappa,\, \pm\rho}(2\varpi r) \approx \Gamma(1 \pm 2\rho)(2\varpi r)^{\frac{1}{2}}(\pm\kappa)^{\mp\rho} J_{\pm 2\rho}(\sqrt{\pm\alpha r}) \quad (37)$$

where $\alpha = 8\kappa\varpi \approx 12M\mu^2$. Consequently, we have

$$
\begin{aligned}
f(\varpi) \approx & \frac{\Gamma(1+2\rho)\Gamma(1-2\rho) r'_* r_*}{2\rho} \left[J_{2\rho}(\sqrt{\alpha r'_*}) J_{-2\rho}(\sqrt{\alpha r_*}) - I_{2\rho}(\sqrt{\alpha r'_*}) I_{-2\rho}(\sqrt{\alpha r_*}) \right] \\
& + \frac{1}{2\rho} \frac{\Gamma(1+2\rho)^2 \Gamma(-2\rho) \Gamma(\frac{1}{2}+\rho-\kappa) r'_* r_*}{\Gamma(2\rho)\Gamma(\frac{1}{2}-\rho-\kappa)} \kappa^{-2\rho} \left[J_{2\rho}(\sqrt{\alpha r'_*}) J_{2\rho}(\sqrt{\alpha r_*}) \right. \\
& \left. + I_{2\rho}(\sqrt{\alpha r'_*}) I_{2\rho}(\sqrt{\alpha r_*}) \right] , \qquad (38)
\end{aligned}
$$

where $I_{\pm 2\rho}$ is the modified Bessel functions. Clearly, the late time tail arising from the first term will be $\sim t^{-1}$. Now let us try to figure out what behavior the second term gives rise to. For this purpose, we define

$$
A = \frac{1}{2\rho} \frac{\Gamma(1+2\rho)^2 \Gamma(-2\rho) r'_* r_*}{\Gamma(2\rho)} \left[J_{2\rho}(\sqrt{\alpha r'_*}) J_{2\rho}(\sqrt{\alpha r_*}) + I_{2\rho}(\sqrt{\alpha r'_*}) I_{2\rho}(\sqrt{\alpha r_*}) \right] , \qquad (39)
$$

then the contribution from the second term to the Green's function can be written as

$$
\frac{A}{2\pi} \int_{-\mu}^{\mu} \frac{\Gamma(\frac{1}{2}+\rho-\kappa)}{\Gamma(\frac{1}{2}-\rho-\kappa)} \kappa^{-2\rho} \, e^{-iwt} , \qquad (40)
$$

which can be approximated, in the limit $\kappa \gg 1$, by using

$$
\Gamma(z)\Gamma(-z) = -\frac{\pi}{z \sin \pi z} , \qquad (41)
$$

and Eq. (6.1.39) in Ref. [17], as follows

$$
\frac{A}{2\pi} \int_{-\mu}^{\mu} e^{i(2\pi\kappa - wt)} \, e^{i\phi} \, dw . \qquad (42)
$$

Here the phase ϕ is defined by

$$
e^{i\phi} = \frac{1 + (-1)^{2\rho} \, e^{-i2\pi\kappa}}{1 + (-1)^{2\rho} \, e^{i2\pi\kappa}} . \qquad (43)
$$

The integral Eq. (42) is very similar to that of Eq. (61) in Ref. [7] and it can be evaluated by method of the saddle-point integration as in such. Hence the asymptotic late time tail arising from the second term is $\sim t^{-\frac{5}{6}}$, and it dominates over the tail from the first term. So, we have

$$
G^C(r'_*, r_*; t) \sim t^{-\frac{5}{6}} . \qquad (44)
$$

IV. SUMMARY

We have studied analytically both the intermediate and asymptotically late-time evolution of massive scalar fields in the background of a black hole with a global monopole. We find that if $\mu M \ll 1$ the intermediate tails given by Eq. (33) dominates at the intermediate late-time $\mu M \ll \mu t \ll 1/(\mu M)^2$ at a fixed radius. Because of the presence of the solid deficit angle in the background metric, the decay is faster than those in the Schwarzschild

and Reissner-Nordström backgrounds [6–8]. Therefore, in the intermediate late-times, the oscillatory power-law tail depends not only on the multiple number of the wave mode but also on the the parameter (b) characterizing the space-time metric. Hence although the intermediate late-time tail is not affected significantly by the curvature, it is by the topology of the background metric. However, the intermediate late-time tail is not the final pattern and a transition to an oscillatory tail with the decay rate of $t^{-5/6}$ is to occur when $\mu t \gg 1/(\mu M)^2$. The origin of the tail may be attributed to the resonance backscattering off the space-time curvature. It is interesting to note that this tail behavior is independent of the field mass, the multiple moment of the wave mode and the space-time parameter b and it is same as that in the black hole backgrounds without global monopoles studied in [7,8]. It should be pointed out, however, that this late time tail begins to dominate only when $\mu t \gg 1/(\mu M)^2$ i.e. $\kappa \gg 1$. So, the tail of massive scalar fields will still be dependent on the multiple moment of the wave mode and the topology of the space-time during the transitional intermediate times when this condition is not satisfied. Our result seems to suggest that the oscillatory $t^{-5/6}$ tail may be a quite general feature for the late-time decay of massive scalar fields in any static black hole backgrounds.

ACKNOWLEDGMENTS

This work was supported by the National Science Foundation of China under Grant 10075019.

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